TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2019

MATHEMATICS: PAPER I

EXAMINATION NUMBER

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 32 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.

2. Read the questions carefully.

3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**

4. Four blank pages (pages 29 to 32) have been included at the end of the exam paper. If you run out of space for a question, use these pages. If you use this extra space, make sure that you indicate this clearly at the question to ensure that your answer is marked in full.

5. Diagrams are not necessarily drawn to scale.

6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated. Ensure that your calculator is in **DEGREE** mode.

7. Clearly show **ALL** calculations, diagrams, graphs etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**

8. Round off to one decimal place unless otherwise stated.

9. It is in your own interest to write legibly and to present your work neatly.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	TOTAL
14	16	15	17	11	9	9	16	22	7	14	/150

SECTION A

QUESTION 1

(a) -2 is one root of the equation $2x^2 + x + k = 0$.

(1) Prove that $k = -6$.

(2)

(2) Determine the other root.

(2)

(b) Solve for x in each case:

(1) $x - 3\sqrt{x+2} = 2$

(6)

(2) $x^2 - x \leq 6$

(4)

[14]

QUESTION 2

Busi opens a new credit card account that charges compound interest at 12,3% p.a. compounded weekly.

Interest:

- 12,3% p.a. compounded weekly

[<https://www.postoffice.co.uk/credit-card/platinum>]

Note: For the calculations in this question, assume that the relevant years have 52 weeks each.

She purchases a computer for an amount of R12 349,00 immediately after activating her credit card.

(a) Show that the balance owing on the credit card one week after the purchase will be R12 378,21 (to the nearest cent).

(3)

(b) Determine how long it will take Busi to pay off the money that she owes on her credit card if she repays R94,75 per week and she does not make any other purchases using this card.

(4)

(c) If the depreciation rate of her computer is 20% per annum on a straight-line basis, determine what its value will be after two years.

(3)

(d) She wants to sell the computer after two years. Will the depreciated value of the computer be sufficient to pay off the outstanding balance immediately after the 104^{th} payment? Show all working.

(6)
[16]

QUESTION 3

(a) (1) Determine $f'(x)$ from first principles if: $f(x) = -5x^2 + x$.

(5)

(2) Hence, or otherwise, determine the equation of the tangent to $f(x)$ at the point where $x = 1$.

(3)

(b) Determine:

(1) $\dfrac{dy}{dx}$ if $y = \dfrac{x^3 + \sqrt{x^3}}{x}$

(4)

(2) $D_x\left[\left(8x^3 - 27\right)\left(4x^2 + 6x + 9\right)^{-1}\right]$

(3)
[15]

QUESTION 4

(a) A pentagon is created using candles as shown in the diagram below.

By adding more candles, a row of two pentagons is formed.

Continuing to add candles, a row of three pentagons can be formed.

If this pattern continues, what is the maximum number of pentagons that can be formed in a row if a total of 100 candles are available?

(4)

(b) An arithmetic series has a first term of 3, a last term of 47 and the sum of all the terms is 300.

(1) Determine the number of terms in the series.

(3)

(2) Determine the common difference.

(3)

(c) Calculate: $\sum_{n=2}^{\infty} 4.\left(\frac{1}{2}\right)^{n+2}$

(4)

(d) In a geometric sequence, the third term is $5p+1$, the fifth term is 4 and the seventh term is 1. Determine the value of p.

(3)
[17]

QUESTION 5

The table below shows the number of passengers that were on a bus after every stop.

	First stop	**Second stop**	**Third stop**	**Fourth stop**
# Passengers	2	20	34	44

The number of passengers on the bus after the n^{th} bus stop can be given by $T_n = an^2 + bn + c$ where a, b and $c \in \mathbb{R}$.

(a) Write down the number of passengers on the bus after the fifth stop.

(1)

(b) Determine a, b and c.

(4)

(c) If it is given that $T_n = -2n^2 + 24n - 20$, determine the maximum number of passengers on the bus.

(3)

(d) Explain why the formula given in Question 5(c) does not work after the eleventh stop.

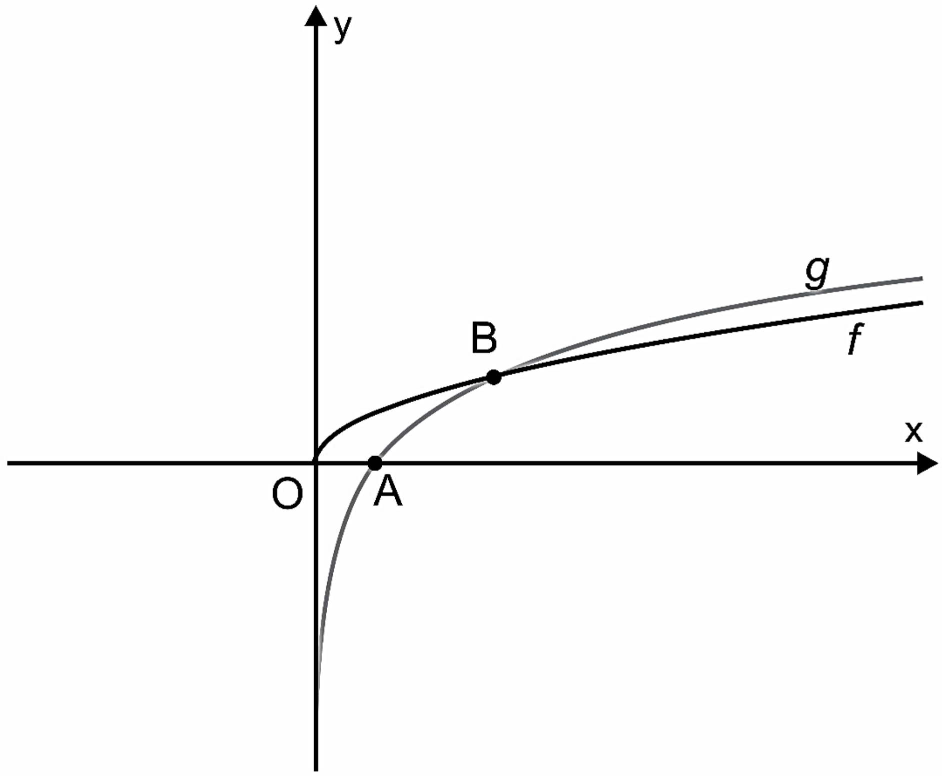
(3)
[11]

73 marks

SECTION B

QUESTION 6

In the diagram below, the graphs of $f(x)=\sqrt{kx}$ and $g(\text{x})=\log_a x$ are given.
Note: O represents the origin.



The graphs of *f* and *g* intersect at the point $\text{B}(3;1)$.

(a) Determine the values of *x*, represented on this sketch, for which $f(x)>g(x)$.

(2)

(b) Determine the values of k and a.

(4)

(c) Determine f^{-1}, the inverse of f in the form $y = ...$, and state its domain.

(3)

[9]

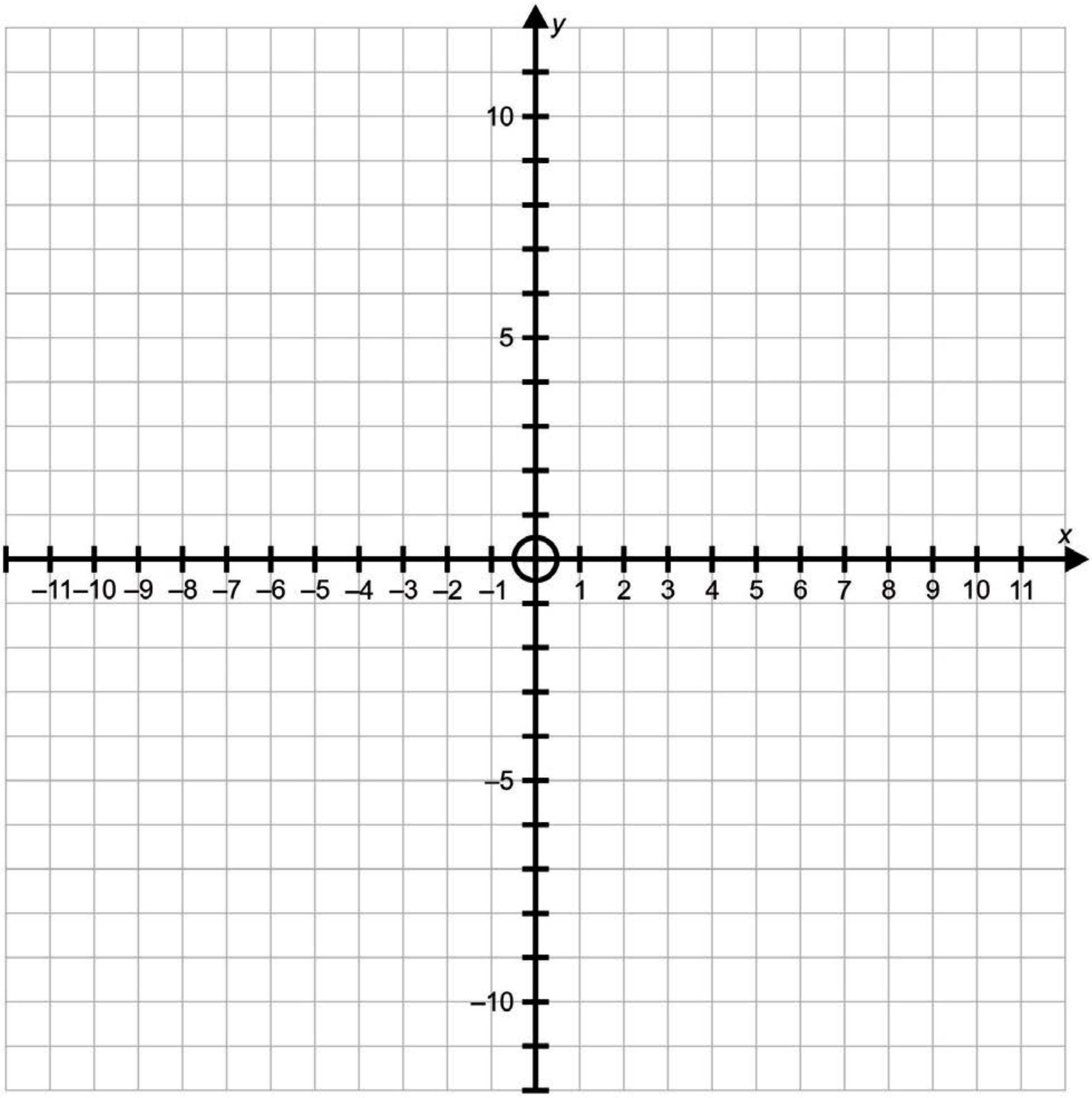
QUESTION 7

(a) Sketch the graphs of $f(x) = 3^{x+1}$ and $g(x) = 3^{2x}$ on the same set of axes.

Show any asymptotes, intercepts with the axes and points of intersection clearly.

Sketch your graph on the grid provided on the next page.

Working space:



(6)

(b) Rewrite the equation $a^{2x} = 3^{x+1}$ in the form $x = ...$

(3)

[9]

QUESTION 8

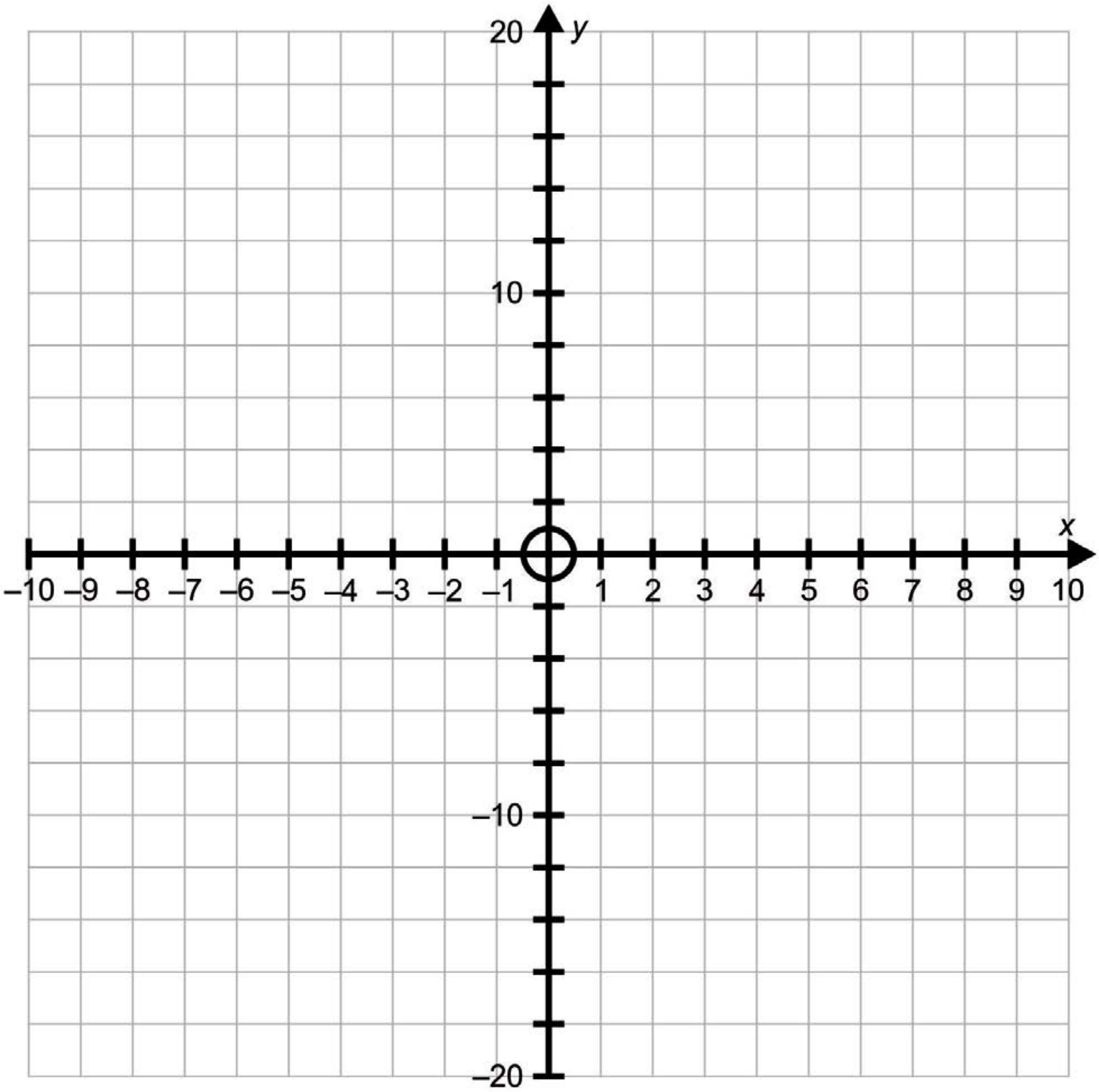
(a) (1) Explain why the equation $(2x-1)^2=-5$ does not have any real solutions.

(1)

(2) On a set of axes, sketch the graph of $y=(2x-1)^2+5$.
Show the coordinates of the turning point and the y-intercept.

Sketch your graph on the grid provided on the next page.

Working space:



(3)

(3) Describe how you would shift the graph vertically so that the x-intercepts are real and equal.

____

____

(1)

(4) Solve for x in terms of k if $(2x-1)^2 = k$ and write down the values of k for which the equation has real roots.

(3)

(5) Give **three** values of k for which the x-intercepts of $y = (2x-1)^2 + k$ will be real, rational and unequal.

(4)

(b) Let the larger root of $px^2 + qx + r = 0$ be P.
Let the larger root of $x^2 + qx + pr = 0$ be Q.

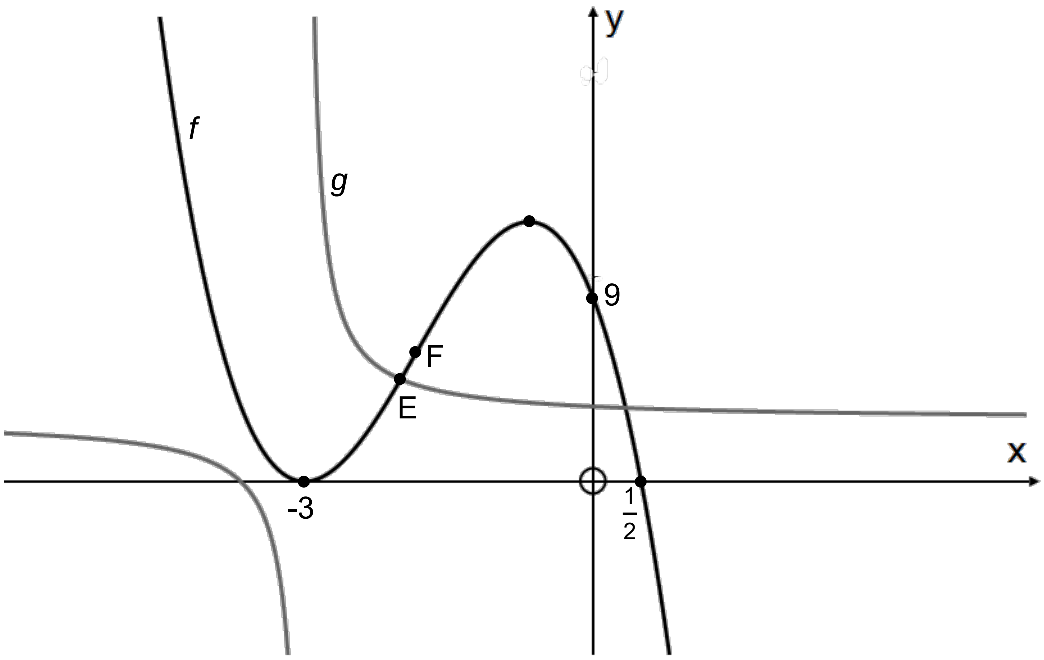
Determine the ratio $P : Q$.

(4)
[16]

QUESTION 9

In the diagram below, the graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = \dfrac{2}{x+p} + q$ are given.

E is a point of intersection of the graphs of *f* and *g*.
F is the point of inflection of *f*.



The graph of *f* cuts the *x*-axis at $x = \dfrac{1}{2}$, touches it at $x = -3$ and cuts the *y*-axis at 9.

(a) (1) Show that $a = -2$, $b = -11$, $c = -12$ and $d = 9$.

(5)

(2) Determine the x co-ordinate of F.

(3)

(b) If the gradient at point E of the graph of $f(x)$ is 8, determine the co-ordinates of E.

(3)

(c) If the graph of g has a vertical asymptote at the minimum stationary point of f, determine the equation of g in the form $y = \dfrac{2}{x+p} + q$.

(3)

(d) Determine the equation of the axis of symmetry of the graph of g that has a positive gradient.

(2)

(e) Determine the value(s) of x for which $f(x) \geq g(x)$ in the interval $x \in (-\infty;0]$.

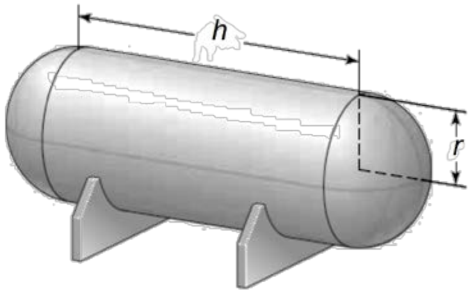
(3)

(f) Determine the values of k, if the graph of f is shifted so that the new graph $h(x) - k = -2x^3 - 11x^2 - 12x + 9$ does not intersect the graph of g for $x \geq 0$.

(3)
[22]

QUESTION 10

An oil tank's structure, as shown in the diagram below, consists of a cylindrical body of length h m and two hemispherical ends of radius r m and has a volume of 1 000 m^3.



[<https://www.chegg.com>]

Determine the value of r such that the total surface area of the tank is a minimum.

Show all working and justifications.

Formulae:

Surface area of sphere $= 4\pi r^2$

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Surface area of cylinder $= 2\pi r^2 + 2\pi rh$

Volume of cylinder $= \pi r^2 h$

[7]

QUESTION 11

(a) Ten coins are arranged in a row:

- five are R1 coins
- three are R2 coins
- two are R5 coins

How many different arrangements are possible, knowing that all the coins of the same value are identical?

(3)

(b) The trees in an orange orchard are harvested twice a year. During the first harvest, 70% of the oranges are picked while the rest are left.

At the second harvest, 35% of the remaining oranges are picked while the rest are not picked.

Assume no oranges were added between harvests.

(1) Calculate the probability that a randomly selected orange will not be picked.

(3)

(2) If it is further given that all the oranges that are picked are packaged with:

- 9% from each harvest selected for export
- 31% sold to the local market and
- the rest are sent to a factory to be made into juice.

What percentage of oranges will be sent to the factory to be made into juice?

(4)

(3) There are 120 oranges in an export box. If 172 export boxes are produced, then how many oranges were there in the total crop?

(4)

[14]

77 marks

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

