IEB
Mathematics
Paper 1
FORM 4
Friday 7 June 2019
Session 1

TIME: 2 hours **TOTAL: 110 marks**

NAME AND SURNAME: Memo version 3.

MATHEMATICS TEACHER:

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 16 pages. Formulae are given at the bottom of this page. Please check that your question paper is complete. The last two pages may be used for working.
- Answer all questions on your question paper.
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- All necessary working which you have used in determining your answers **must** be clearly shown.
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to **2 decimal places** unless otherwise stated.
- Ensure that your calculator is in DEGREE mode.
- Diagrams have not necessarily been drawn to scale.
- State all restrictions where necessary.

ME Q1-Q2.3 AG Q2.4-Q5

Questions	1	2	3	4	5	TOTAL
Out of	34	29	18	20	9	110
Mark awarded	34	29	18	20	9	110

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

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SECTION A

QUESTION 1 [34]

Solve for x **without using a calculator**. You can use the calculator to check your answers. Leave answers in surd form if necessary and with positive exponents.

a) $8^{-x} = \frac{1}{64}$ (4)

$2^{-3x} = 2^{-6}$ a a

$-3x = -6$ ca

$x = 2$ ca

b) $(9)^{x^2-9} = 1$ (4)

$(3^2)^{x^2-9} = 3^0$ a a

$2x^2 - 18 = 0$ ca

$x^2 = 9$

$x = 3$ or $x = -3$ ca

c) $\frac{1}{4}x^{\frac{3}{2}} = 2$ (4)

$x^{\frac{3}{2}} = 8$ a

$(x^{3/2})^{2/3} = (2^3)^{2/3}$ ca ma

$x = 2^2$ ca

$x = 4$

12

d) $6 + \frac{-6}{-x-3} = \frac{2x}{x+3} + 3x$ (7)

$6 + \frac{6}{x+3} = \frac{2x}{x+3} + 3x$ LCD: $x+3$ ✓a

$6(x+3) + 6 = 2x + 3x(x+3)$ ✓ma ✓a

$6x + 18 + 6 = 2x + 3x^2 + 9x$ ✓a

$0 = 3x^2 + 5x - 24$ ✓m quad form

$x = \frac{-5 + \sqrt{313}}{6}$ ✓ca or $x = \frac{-5 - \sqrt{313}}{6}$ ✓ca

2,115 or −3,78.

* Have added more space in final copy.

for answering $\frac{4}{3}$ or −3 4/7

$\frac{4}{3}$ or −3 5/7

e) $\sqrt{4-x} + x = -2$ (5)

$\sqrt{4-x} = -2 - x$

$4 - x = (-2-x)^2$ ✓ma

$4 - x = 4 + 4x + x^2$ ✓a

$0 = x^2 + 4x + x$

$0 = x^2 + 5x$ ✓ca

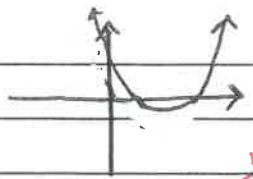
$0 = x(x+5)$

$x \neq 0$ ✓a or $x = -5$ ✓a

f) $x^2 - 6x \geq -8$ (4)

$x^2 - 6x + 8 \geq 0$

$(x-2)(x-4) \geq 0$ ✓a



✓ma

$x \geq 4$ or $x \leq 2$

✓ca ✓ca

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g) Solve for x and y given

$2x - y = 1$ and $x^2 - yx = 3x - 3$ ...(B)

(6)

$\Rightarrow y = 2x - 1$ ...(A)

(A) into (B): $x^2 - (2x-1)x = 3x - 3$ ma

$x^2 - 2x^2 + x = 3x - 3$

$+x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$ a

$x = -3$ or $x = 1$ ca ...(C)

(C) into (A): $y = 2(-3) - 1$ ma $\quad y = 2(1) - 1$

$= -7$ ca $\quad = 1$ ca

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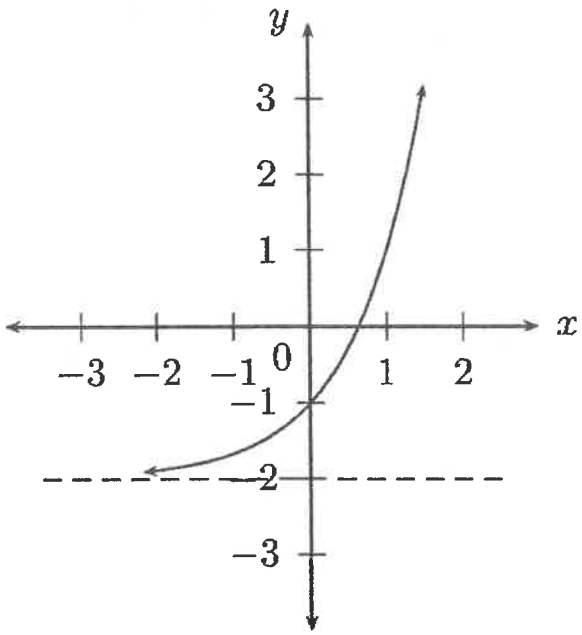
QUESTION 2 [29]

2.1. Determine the equations of the following graphs:

a) $y = 2^{x-p} + q$ (4)



$y = 2^{x-p} - 2$ ✓a

Sub $(0;-1)$

$-1 = 2^{-p} - 2$ ✓ma

$1 = 2^{-p}$

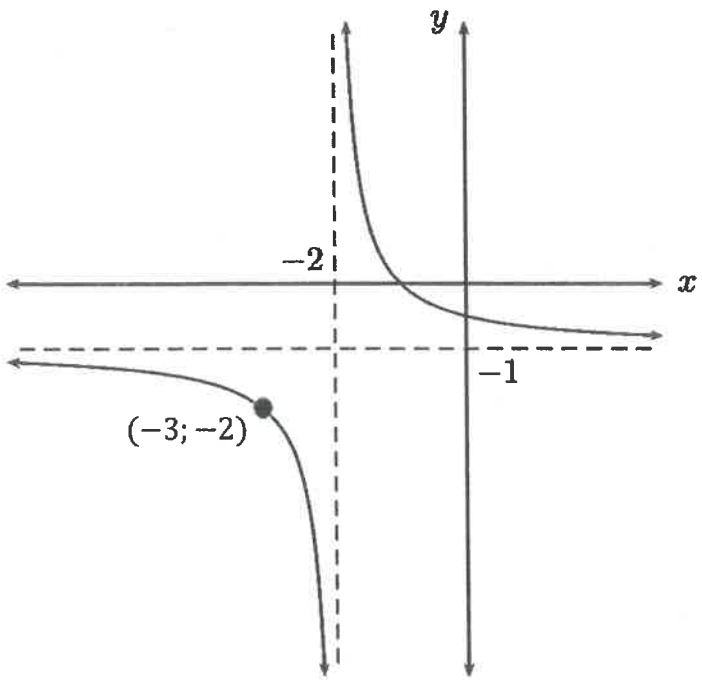
$2^0 = 2^{-p}$ ✓ca

$0 = -p$

$0 = p$ ✓ca

$y = 2^x - 2$

b) $y = \frac{a}{x-p} + q$ (4)



$y = \frac{a}{x+2} - 1$ ✓a ✓a

Sub $(-3;-2)$:

$-2 = \frac{a}{-3+2} - 1$ ✓ma

$-1 = \frac{a}{-1}$

$1 = a$ ✓ca

$y = \frac{1}{x+2} - 1$

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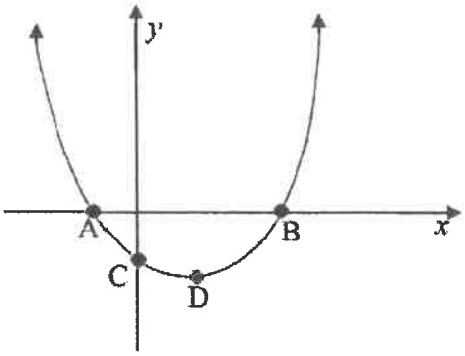
2.2. The graph of $f(x) = 3(x+1)(x-3)$, not drawn to scale is sketched below.



a) Determine the **coordinates** of A, B, C and D.

(6)

A(−1;0) ✓a B(3;0) ✓a

✓ma or $x = \frac{-b}{2a}$ etc
(using symmetry)

$y = 3(1+1)(1-3)$ ✓a

$y = -12$

$\therefore D(1;-12)$ ✓ca

$C(0;-9)$ ✓a

b) If $f(x) = k$, for which values of k will you get two real solutions?

(2)

$k > -12$ ✓a ✓ca

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c) For which values of x will $f(x)$ be decreasing?

(2)

$x < 1$ ✓a ✓ca .

d) What is the minimum value of $f(x)$?

(1)

$f(x) = -12$ ✓a min value $= -12$

2.3. Determine the new equation of $f(x) = 4x^2 + 5x + 1$ and $g(x) = \frac{2}{x+5} - 6$ *if*:

a) $f(x)$ is reflected about the y-axis.

(2)

$y = 4(-x)^2 + 5(-x) + 1$ ✓a

$y = 4x^2 - 5x + 1$ ✓ca

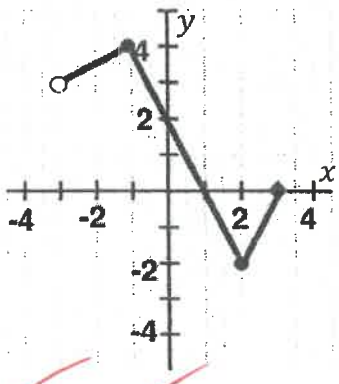
b) $g(x)$ is shifted 1 unit right and 4 units down.

(2)

$y = \frac{2}{x+4}$ ✓a $- 10$ ✓a

2.4. Give the range and domain of the following graph.

(4)



$x \in (-3; 3]$ ✓ ✓ $y \in [-2; 4]$ ✓ ✓

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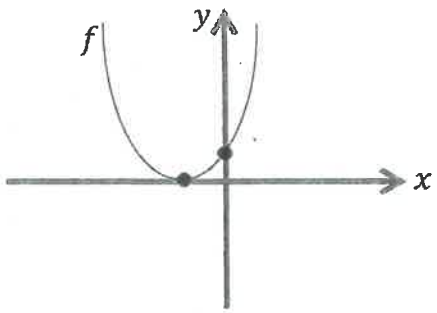
2.5. The sketch shows the graph defined by $f(x) = ax^2 + bx + c$. Which of the following statements (***A***, ***B***, ***C*** or ***D***) is completely true? Circle the correct answer.



A $-\frac{b}{2a} > 0, b^2 - 4ac = 0, c > 0$

B $a > 0, b < 0, c > 0$

C $b^2 - 4ac = 0, c < 0, b > 0$

D $b^2 - 4ac = 0, c > 0, b > 0$

(2)

✓ Changed for

✓✓

2

SECTION B

QUESTION 3 [18]

Without using a calculator, simplify:

a) $\dfrac{6^{2m}\times 4^{-m}}{(3^m)^2}$ (4)

$$\frac{(2\cdot 3)^{2m}\cdot (2^2)^{-m}}{3^{2m}}$$ ma

$$= \frac{2^{2m}\cdot 3^{2m}\cdot 2^{-2m}}{3^{2m}}$$ ca

$$= \frac{2^{2m}}{2^{2m}} = 1$$ ca ca

b) $\dfrac{(2a)^{-2}\times 3a^{-5}}{\left(a^3 b^{\frac{1}{2}}\right)^2\times (ab)^0}$ (5)

$$\frac{2^{-2}a^{-2}\times 3a^{-5}}{a^6\cdot b\cdot 1} = \frac{3}{4a^{13}b}$$ a a ca ca

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c) Simplify by rationalising the denominator (show all working without using a calculator)

$\frac{3}{\sqrt{7}-\sqrt{5}}$ (2)

$$\frac{3}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{3\sqrt{7}+3\sqrt{5}}{2}$$ a a

d) i. Simplify $\frac{3^{x+1}-3^x}{3^x}$ (2)

$$\frac{3^x(3-1)}{3^x} = 2$$ ma ca

ii. Hence or otherwise simplify $\frac{3^{x+1}-3^x}{m.3^x+2^2 3^x} - \frac{3m-12}{m^2-16}$ (5)

$$\frac{3^x(3-1)}{3^x(m+4)} - \frac{3(m-4)}{(m+4)(m-4)}$$ a a

$$= \frac{2}{m+4} - \frac{3}{m+4} = \frac{-1}{m+4}$$ ca ca

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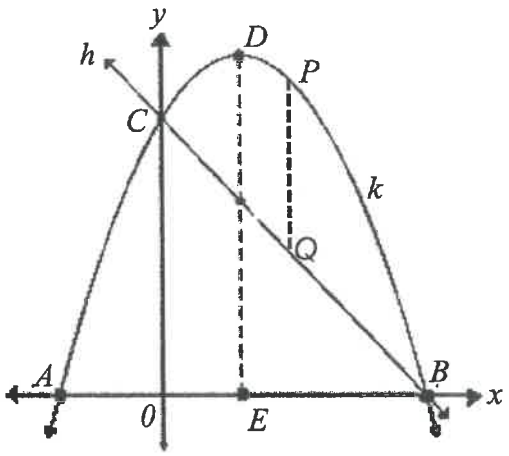
QUESTION 4 [20]

Given: the graph of $k(x) = -x^2 + 3x + 10$ with turning point at D. The graph of the straight line $h(x) = mx + c$ passing through points B and C is also shown.



a) Determine the lengths A0, 0B, 0C and DE (8)

$y = -x^2 + 3x + 10$ a

$0 = -x^2 + 3x + 10$ ca

$0 = (x+2)(x-5)$

$x = -2$ or $x = 5$

$\therefore$ A0 = 2 units ca OB = 5 units ca

OC = 10 units a

$x = \frac{-(3)}{2(-1)} = \frac{3}{2}$ a $y = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 10$ a

$= \frac{49}{4}$

DE = $\frac{49}{4}$ units ca

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b) Determine the equation of $h(x)$ (3)

$y = mx + 10$ ca

$y = -\frac{10}{5}x + 10$ ca

$h(x) = -2x + 10$ ca

c) Determine the x-values for which $k(x) > 0$ (2)

$-2 < x < 5$ ca ca

d) Determine the x-values for which $k(x) \geq h(x)$ (2)

$0 \leq x \leq 5$ ca ca

$-x^2 + 3x + 10 \geq -2x + 10$

$-x^2 + 5x \geq 0$

e) If P is any point on $k(x)$ and Q any point on $h(x)$ such that PQ is a vertical distance, determine the maximum length of PQ. (5)

$PQ = -x^2 + 3x + 10 - (-2x + 10)$ ca

$= -x^2 + 5x$ ca

$x = \frac{-5}{2(-1)} = \frac{5}{2}$ ca

$PQ = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right)$ ca

$= \frac{25}{4} = 6{,}25$ units ca

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QUESTION 5 [9]

a) If $f(x) = a(x-2)^2 + c$ and $g(x) = (2x-5)(x-b)$ are functions representing the same parabola, then find the value of a, b and c. (7)

$f(x) = a(x^2 - 4x + 4) + c$ ma

$f(x) = ax^2 - 4ax + 4a + c$

$g(x) = 2x^2 - 2xb - 5x + 5b$

$\Rightarrow a = 2$ a

$-4a = -2b - 5$ a

$-4(2) = -2b - 5$ ma

$\frac{3}{2} = b$ ca

$4a + c = 5b$ a

$4(2) + c = 5\left(\frac{3}{2}\right)$

$c = -\frac{1}{2}$ ca

b) A certain company sells x units of a product.

Its profit in rands is given by: $P(x) = -3(x-17)^2 + 560$.

Write down the number of units that must be sold to earn a maximum profit, as well as the maximum profit. (2)

no of units = 17 a

max price = R560 a

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EXTRA PAPER FOR WORKING IF NEEDED