basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2012

MEMORANDUM

MARKS: 150

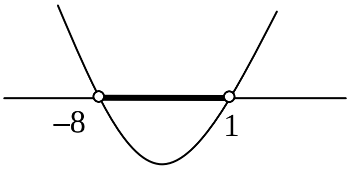
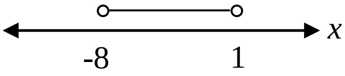
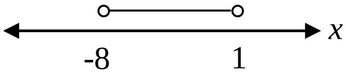
This memorandum consists of 30 pages.

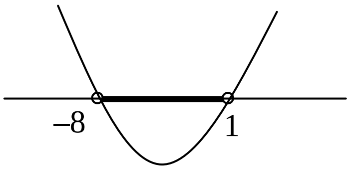
NOTE:

- If a candidate answered a question TWICE, mark the FIRST attempt ONLY.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed out question.
- Consistent accuracy applies in ALL aspects of the memorandum.

QUESTION 1

1.1.1	$(2x-1)(x+4)=0$ $x=\frac{1}{2}$ or -4	✓ answer ✓ answer (2)
1.1.2	$3x^2-x=5$ $3x^2-x-5=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-5)}}{2(3)}$ $=\frac{1\pm\sqrt{61}}{6}$ $=1{,}47$ or $-1{,}14$ **Note:** if a candidate uses incorrect formula award max 1 mark (for standard form) **Note:** if a candidate has not rounded off correctly, penalise 1 mark **OR** $3x^2-x=5$ $x^2-\frac{1}{3}x=\frac{5}{3}$ $\left(x-\frac{1}{6}\right)^2=\frac{5}{3}+\frac{1}{36}$ $\left(x-\frac{1}{6}\right)=\pm\sqrt{\frac{61}{36}}$ $x=\frac{1}{6}\pm\sqrt{\frac{61}{36}}$ $=1{,}47$ or $-1{,}14$ **OR**	✓ standard form ✓subs into correct formula ✓✓answer (4) ✓ division by 3 ✓ $\left(x-\frac{1}{6}\right)=\pm\sqrt{\frac{61}{36}}$ ✓✓answer (4)

	$3x^2 - x = 5$ $3x^2 - x - 5 = 0$ $x^2 - \frac{x}{3} - \frac{5}{3} = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-\left(-\frac{1}{3}\right) \pm \sqrt{\left(-\frac{1}{3}\right)^2 - 4(1)\left(-\frac{5}{3}\right)}}{2(1)}$ $= \frac{\frac{1}{3} \pm \sqrt{\frac{61}{9}}}{2}$ $= 1{,}47$ or $-1{,}14$	✓ standard form ✓subs into correct formula ✓✓answer (4)
1.1.3	$x^2 + 7x - 8 < 0$ $(x+8)(x-1) < 0$ $\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & -8 & & 1 & \end{array}$ **OR**  **OR** $\begin{array}{lccccc} x & & -8 & & 1 & \\ \hline x+8 & - & 0 & + & + & + \\ x-1 & - & - & - & 0 & + \\ (x+8)(x-1) & + & 0 & - & 0 & + \end{array}$ Therefore the solution is: $-8 < x < 1$ **OR** $x \in (-8\,;1)$ **OR**  **OR** $x^2 + 7x - 8 < 0$ $(x+8)(x-1) < 0$ $\therefore\ x+8 < 0$ and $x-1 > 0$ or $x+8 > 0$ and $x-1 < 0$ $x < -8$ and $x > 1$ $x > -8$ and $x < 1$ No solution Therefore the solution is: $-8 < x < 1$ **OR** $x \in (-8\,;1)$ **OR**  	✓ factors ✓ $-8,\ 1$ ✓✓ answer (4) ✓ factors ✓ $-8,\ 1$ ✓✓ answer (4)

	NOTE: **In this alternative, award max 3/4 marks since there is no conclusion** $x^2 + 7x - 8 < 0$ $(x+8)(x-1) < 0$ 	✓ factors ✓ – 8, 1 ✓ graph with bolded line
1.2.1	$4y - x = 4$ and $xy = 8$ $x = 4y - 4$ $(4y-4)y = 8$ $(y-1)y = 2$ $y^2 - y - 2 = 0$ $(y+1)(y-2) = 0$ $y = -1$ or $y = 2$ $x = -8$ or $x = 4$ (x ; y) = (– 8 ; – 1) or (4 ; 2) **Note:** If candidate makes a mistake which leads to both equations being LINEAR award maximum 2/6 marks	✓ $x = 4y - 4$ ✓ substitution ✓ factors ✓ y-values ✓✓ x-values (6)
	OR $4y - x = 4$ and $xy = 8$ $x = 4y - 4$ $(4y-4)y = 8$ $(y-1)y = 2$ By inspection $y = -1$ or $y = 2$ $x = -8$ or $x = 4$ (x ; y) = (– 8 ; – 1) or (4 ; 2)	✓ $x = 4y - 4$ ✓ substitution ✓✓ y-values ✓✓ x-values (6)
	OR $4y - x = 4$ and $xy = 8$ $x = 4y - 4$ $(4y-4)y = 8$ $(y-1)y = 2$ $y^2 - y - 2 = 0$ $y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ $y = -1$ or $y = 2$ $x = -8$ or $x = 4$ (x ; y) = (– 8 ; – 1) or (4 ; 2)	✓ $x = 4y - 4$ ✓ substitution ✓ subs into correct formula ✓ y-values ✓✓ x-values (6)

	OR $4y - x = 4$ and $xy = 8$ $y = \frac{x}{4} + 1$ $x\left(\frac{x}{4} + 1\right) = 8$ $\frac{x^2}{4} + x - 8 = 0$ $x^2 + 4x - 32 = 0$ $(x + 8)(x - 4) = 0$ $x = -8$ or $x = 4$ $y = -1$ or $y = 2$ $(x ; y) = (-8 ; -1)$ or $(4 ; 2)$	✓ $y = \frac{x}{4} + 1$ ✓ substitution ✓ factors ✓ x-values ✓✓ y-values (6)
	OR $4y - x = 4$ and $xy = 8$ $y = \frac{x}{4} + 1$ $x\left(\frac{x}{4} + 1\right) = 8$ $\frac{x^2}{4} + x - 8 = 0$ $x^2 + 4x - 32 = 0$ $x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-32)}}{2(1)}$ $x = -8$ or $x = 4$ $y = -1$ or $y = 2$ $(x ; y) = (-8 ; -1)$ or $(4 ; 2)$	✓ $y = \frac{x}{4} + 1$ ✓ substitution ✓ subs into correct formula ✓ x-values ✓✓ y-values (6)
	OR $xy = 8$ and $4y - x = 4$ $x = \frac{8}{y}$ $4y - \frac{8}{y} = 4$ $4y^2 - 4y - 8 = 0$ $y^2 - y - 2 = 0$ $(y - 2)(y + 1) = 0$ $y = -1$ or $y = 2$ $x = -8$ or $x = 4$ $(x ; y) = (-8 ; -1)$ or $(4 ; 2)$	✓ $x = \frac{8}{y}$ ✓ substitution ✓ factors ✓ y-values ✓✓ x-values (6)

	OR	
	$xy = 8$ and $4y - x = 4$	
	$x = \frac{8}{y}$	✓ $x = \frac{8}{y}$
	$4y - \frac{8}{y} = 4$	✓ substitution
	$4y^2 - 4y - 8 = 0$	
	$y^2 - y - 2 = 0$	
	$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$	✓ subs into correct formula
	$y = -1$ or $y = 2$	✓ *y*-values
	$x = -8$ or $x = 4$	✓✓ *x*-values
	$(x ; y) = (-8 ; -1)$ or $(4 ; 2)$	(6)
	OR	
	$xy = 8$ and $4y - x = 4$	
	$y = \frac{8}{x}$	✓ $y = \frac{8}{x}$
	$4\left(\frac{8}{x}\right) - x = 4$	✓ substitution
	$0 = x^2 + 4x - 32$	
	$0 = (x + 8)(x - 4)$	✓ factors
	$x = -8$ or $x = 4$	✓ *x*-values
	$y = -1$ or $y = 2$	✓✓ *y*-values
	$(x ; y) = (-8 ; -1)$ or $(4 ; 2)$	(6)
	OR	
	$xy = 8$ and $4y - x = 4$	
	$y = \frac{8}{x}$	✓ $y = \frac{8}{x}$
	$4\left(\frac{8}{x}\right) - x = 4$	✓ substitution
	$0 = x^2 + 4x - 32$	
	$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-32)}}{2(1)}$	✓ subs into correct formula
	$x = -8$ or $x = 4$	✓ *x*-values
	$y = -1$ or $y = 2$	✓✓ *y*-values
	$(x ; y) = (-8 ; -1)$ or $(4 ; 2)$	(6)

1.2.2	$4x - y = 4$ **OR** $y = 4x - 4$ **OR** $x = \frac{y+4}{4}$ **OR** $4x - y - 4 = 0$ **OR** $x = \frac{1}{4}y + 1$	✓✓ interchanges x and y (2)
1.3.1	$\sqrt{2p+5} = 0$ $2p + 5 = 0$ $2p = -5$ $p = -\frac{5}{2}$	✓ $2p + 5 = 0$ **or** $\sqrt{2p+5} = 0$ **or** $\frac{-2 \pm \sqrt{0}}{7}$ ✓ answer (2)
1.3.2	$2p + 5 < 0$ $p < -\frac{5}{2}$	✓ answer (1) **[21]**

QUESTION 2

2.1	$T_2 - T_1 = T_3 - T_2$ $2x - (3x+1) = (3x-7) - 2x$ $2x - 3x - 1 = 3x - 7 - 2x$ $-x - 1 = x - 7$ $-2x = -6$ $x = 3$ **OR** $T_2 = \frac{T_1 + T_3}{2}$ $2x = \frac{(3x+1)+(3x-7)}{2}$ $4x = 6x - 6$ $6 = 2x$ $x = 3$ **OR** $T_3 - T_1 = 2(T_2 - T_1)$ $(3x-7) - (3x+1) = 2(2x - (3x+1))$ $-8 = -2x - 2$ $2x = 6$ $x = 3$	✓ $T_2 - T_1 = T_3 - T_2$ or $2x - (3x+1) = (3x-7) - 2x$ ✓ answer (2) ✓ $T_2 = \frac{T_1 + T_3}{2}$ or $2x = \frac{(3x+1)+(3x-7)}{2}$ ✓ answer (2) ✓ $T_3 - T_1 = 2(T_2 - T_1)$ or $(3x-7) - (3x+1) = 2(2x - (3x+1))$ ✓ answer (2)
2.2.1	$T_n = a + (n-1)d$ $T_{11} = 10 + (11-1)(-4)$ $= -30$ **OR** 10; 6; 2; –2; –6; –10; –14; –18; –22; –26; –30 … $\therefore T_{11} = -30$	✓$d = -4$ ✓answer (2) ✓expands sequence ✓answer (2)

2.2.2	$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$	✓correct formula ✓substitution of *a* and *d*
	$-560=\frac{n}{2}\left[2(10)+(n-1)(-4)\right]$	✓subs $S_n=-560$
	$-1120=-4n^2+24n$	
	$4n^2-24n-1120=0$	✓ $-4n^2+24n+1120=0$ or $4n^2-24n-1120=0$ or $n^2-6n-280=0$
	$n^2-6n-280=0$	
	$(n-20)(n+14)=0$	✓factors
	$n=20$ or -14	
	$\therefore n=20$ only	✓ selects $n=20$ only
	Note: if candidate substitutes into **incorrect formula**, award 0/6	(6)
	OR	
	$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$	✓ correct formula
	Note: if candidate writes **answer only**, award 1/6 marks	
	$-560=\frac{n}{2}\left[2(10)+(n-1)(-4)\right]$	✓substitution of *a* and *d* ✓subs $S_n=-560$
	$-1120=-4n^2+24n$	
	$4n^2-24n-1120=0$	
	$n^2-6n-280=0$	✓ $4n^2-24n-1120=0$ or $-4n^2+24n+1120=0$ or $n^2-6n-280=0$
	$n=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(-280)}}{2(1)}$	✓subs into correct formula
	$n=20$ or -14	
	$\therefore n=20$ only	✓ selects $n=20$ only
		(6)
	OR	
	$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$	✓ correct formula
	$-560=\frac{n}{2}\left[2(10)+(n-1)(-4)\right]$	✓substitution of *a* and *d* ✓subs $S_n=-560$
	$-560=\frac{20n}{2}-\frac{4n^2}{2}+\frac{4n}{2}$	
	$2n^2-12n-560=0$	✓ $2n^2-12n-560=0$ or $-2n^2+12n+560=0$ or $n^2-6n-280=0$
	$n^2-6n-280=0$	
	$(n-20)(n+14)=0$	✓factors
	$n=20$ or -14	
	$\therefore n=20$ only	✓ selects $n=20$ only
		(6)

<table>
<tr>
<td></td>
<td>OR

$S_{11} = -110$
<table>
<tr><td>n</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr>
<tr><td>T_n</td><td>–34</td><td>–38</td><td>–42</td><td>–46</td><td>–50</td><td>–54</td><td>–58</td><td>–62</td><td>–66</td></tr>
<tr><td>S_n</td><td>–144</td><td>–182</td><td>–224</td><td>–270</td><td>–320</td><td>–374</td><td>–432</td><td>–494</td><td>–560</td></tr>
</table>

$\therefore n = 20$</td>
<td>✓ $S_{11} = -110$

✓ sequence expanded
✓✓ series calculated

✓✓ answer
(6)
[10]</td>
</tr>
</table>

QUESTION 3

<table>
<tr>
<td>3.1.1</td>
<td>$T_n = ar^{n-1}$
$= 27\left(\frac{1}{3}\right)^{n-1}$

Note: The final answer can also be written as 3^{4-n} or $\left(\frac{1}{3}\right)^{n-4}$</td>
<td>✓ $a = 27$ and $r = \frac{1}{3}$
✓ substitute into correct formula
(2)</td>
</tr>
<tr>
<td>3.1.2</td>
<td>$-1 < r < 1$ or $|r| < 1$

Note: If candidate concludes series is not convergent, award 0 marks.

OR
The common ratio (r) is $\frac{1}{3}$ which is between –1 and 1.
OR
$-1 < \frac{1}{3} < 1$</td>
<td>✓ answer
(1)

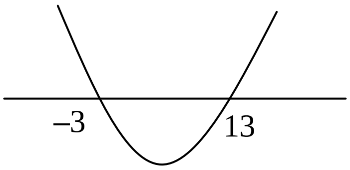
✓ answer
(1)

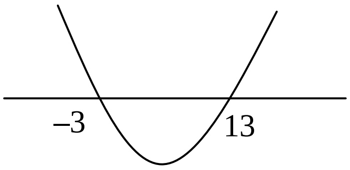
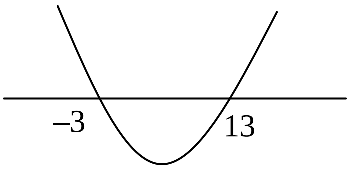
✓ answer
(1)</td>
</tr>
<tr>
<td>3.1.3</td>
<td>$S_\infty = \frac{a}{1-r}$
$= \frac{27}{1-\frac{1}{3}}$
$= \frac{81}{2}$ or 40,5 or 41

Note: If $r > 1$ or $r < -1$ is substituted then 0/2 marks.</td>
<td>✓ substitution

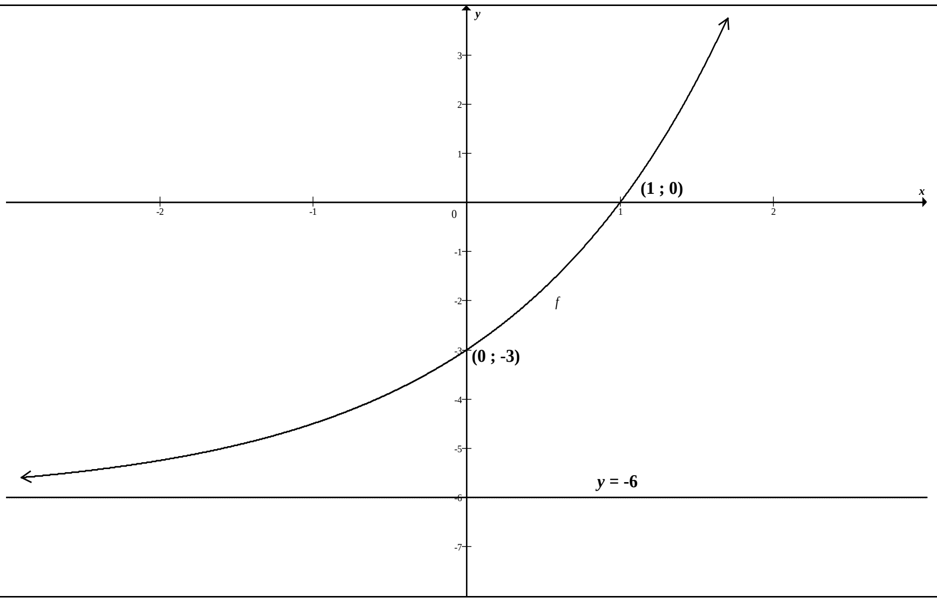
✓ answer
(2)</td>
</tr>
</table>

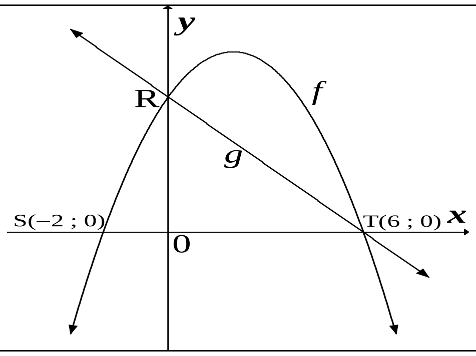
3.2	Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$	✓ $\frac{V}{2}$
	Note: If candidate lets the volume of the first tank be a specific value (instead of a variable) and his/her argument follows correctly, award 4/4 marks **Note:** If candidate answers 'Yes' only with no justification: 1/4 marks	
	$S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$	✓ substitute into correct formula
	$=\frac{524287}{524288}V$ $=0,9999980927\ V$ $<V$	✓ answer
	Yes, the water will fill the first tank without spilling over.	✓ conclusion (4)
	OR	
	Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$	✓ $\frac{V}{2}$
	$S_{19}=\frac{\frac{V}{2}\left[1-\left(\frac{1}{2}\right)^{19}\right]}{1-\frac{1}{2}}$	✓ substitute into correct formula
	$=V\left[1-\left(\frac{1}{2}\right)^{19}\right]$ $<V\cdot 1$ $=V$	✓ observes that $\left[1-\left(\frac{1}{2}\right)^{19}\right]<1$
	Yes, the water will fill the first tank without spilling over.	✓ conclusion (4)
	OR	
	Let V be the volume of the first tank. $\frac{V}{2};\frac{V}{4};\frac{V}{8}.......$	✓ $\frac{V}{2}$
	$S_{\infty}=\frac{\frac{V}{2}}{1-\frac{1}{2}}$ $=V$	✓ substitute into correct formula
	Since the first tank will hold the water from infinitely many tanks without spilling over, certainly: Yes, the first tank will hold the water from the other 19 tanks without spilling over.	✓✓ correct argument (4)

	OR If the tanks are emptied one by one, starting from the second, each tank **will fill only half the remaining space**, so the first tank can hold all the water from the other 19 tanks.	✓ Yes (explicit or understood from the argument.) ✓✓✓ argument (4)
3.3.1	$T_n = -2(n-5)^2 + 18$ Term 1 = – 14 Term 2 = 0 Term 3 = 10	✓ – 14 ✓ 0 ✓ 10 (3)
3.3.2	Term 5 **OR** $n = 5$ **OR** T_5	✓ answer (1)
3.3.3	Second difference = 2*a* Second difference = 2(– 2) Second difference = – 4 **OR** -14 0 10 14 10 – 4 Second difference = – 4	✓ subs – 2 into 2*a* ✓ answer (2) ✓ first differences ✓second difference (2)
3.3.4	$-2(n-5)^2 + 18 < -110$ $-2(n-5)^2 + 128 < 0$ $-2n^2 + 20n - 50 + 128 < 0$ $-2n^2 + 20n + 78 < 0$ $n^2 - 10n - 39 > 0$ $(n-13)(n+3) > 0$ **Note:** Answer only award 2/6 marks + 0 − 0 + -3 13  $n < -3$ or n > 13 $n \geq 14$; $n \in \mathrm{N}$ **OR** $n > 13$; $n \in \mathrm{N}$ **OR**	✓ $T_n < -110$ ✓standard form ✓factors ✓ critical values ✓ inequalities ✓ $n > 13$ (accept: $n \geq 14$) (6)

	$-2(n-5)^2+18<-110$ $-2(n-5)^2+128<0$ $(n-5)^2-64>0$ $\left[(n-5)-8\right]\left[(n-5)+8\right]>0$ $(n-13)(n+3)>0$ + 0 − 0 + -3 13  $n<-3$ or n > 13 $n\geq 14$; $n\in \mathrm{N}$ **OR** $n>13$; $n\in \mathrm{N}$	✓ $T_n<-110$ ✓ $(n-5)^2-64>0$ ✓factors ✓ critical values ✓ inequalities ✓ $n>13$ (accept: $n\geq 14$) (6)
	OR $-2(n-5)^2+18<-110$ $-2(n-5)^2<-128$ $(n-5)^2>64$ $n-5<-8$ or $n-5>8$ $n<-3$ or $n>13$ $n\geq 14$; $n\in \mathrm{N}$ **OR** $n>13$; $n\in \mathrm{N}$	✓ $T_n<-110$ ✓ $2(n-5)^2>128$ ✓ 8 and – 8 ✓ $n-5>8$ ✓ $n-5<-8$ ✓ $n>13$ (accept: $n\geq 14$) (6)
	OR $T_n=-2(n-5)^2+18$ $T_n=-2n^2+20n-32$ $-2n^2+20n-32<-110$ $-2n^2+20n-78<0$ $n^2-10n-39>0$ $(n-13)(n+3)>0$ + 0 − 0 + -3 13  $n<-3$ or $n>13$ $n\geq 14$; $n\in \mathrm{N}$ **OR** $n>13$; $n\in \mathrm{N}$	✓ $T_n<-110$ ✓standard form ✓factors ✓ critical values ✓ inequalities ✓ $n>13$ (accept: $n\geq 14$) (6)
	OR –14 ; 0 ; 10 ; 16 ; 18 ; 16 ; 10 ; 0 ; –14 ; –32 ; –54 ; –80 ; –110 $n\geq 14$; $n\in \mathrm{N}$	✓✓✓✓ expansion ✓✓ conclusion of $n\geq 14$ (accept $n>13$) (6) **[21]**

QUESTION 4

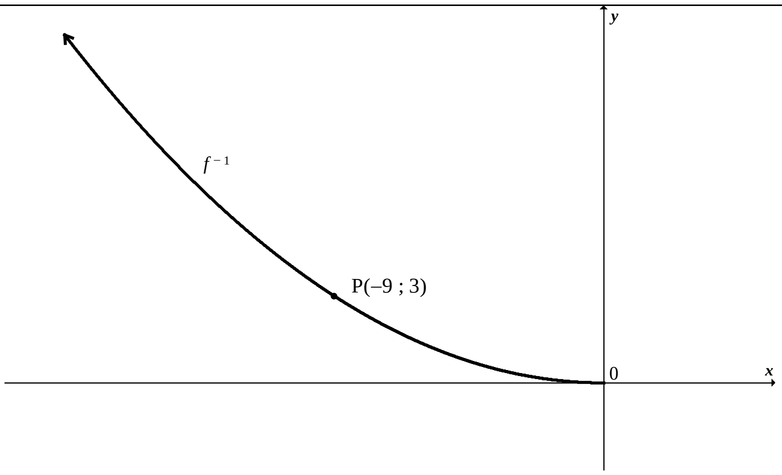
4.1.1	$y = 3.2^0 - 6$ $y = 3 - 6$ $y = -3$ $(0\,;-3)$	✓ answer (1)
4.1.2	$0 = 3.2^x - 6$ $3.2^x = 6$ $2^x = 2^1$ $x = 1$ $(1\,;0)$ **Note:** If a candidate interchanges question 4.1.1 and 4.1.2: 0/3 marks **Note:** If a candidate says that $3.2^x = 6^x$ (i.e. wrong mathematics) s/he will arrive at correct answer BUT award max 1/2	✓ $y = 0$ ✓ x-value (2)
4.1.3		✓intercepts ✓ asymptote ✓ shape (3)
4.1.4	$y > -6$ OR $(-6\,;\infty)$	✓ answer (1)

4.2		
4.2.1	$y=-2x+d$ $0=(-2)(6)+d$ $d=12$ **OR** $y-y_1=m(x-x_1)$ $y-0=-2(x-6)$ $y=-2x+12$ $\therefore d=12$ **OR** Since $m=-2$ and $m=\frac{-d}{6}$ $-2=\frac{-d}{6}$ $d=12$	✓substitution ✓ answer (2) ✓substitution ✓ answer (2) ✓substitution ✓ answer (2)
4.2.2	$y=a(x-6)(x+2)$ $12=a(0-6)(0+2)$ $a=-1$ $y=-(x^2-4x-12)$ $=-x^2+4x+12$ **Note:** No marks for answer only. **OR** $y=ax^2+bx+12$ $0=a(-2)^2+b(-2)+12$ i.e. $0=4a-2b+12$ $0=a(6)^2+b(6)+12$ i.e. $\underline{0=36a+6b+12}$ $0=24b-96$ $b=4$ $0=4a-2(4)+12$ $a=-1$ $y=-x^2+4x+12$	✓ $y=a(x-6)(x+2)$ ✓ subs R(0 ; 12) ✓ a-value ✓ $y=-x^2+4x+12$ (4) ✓ $y=ax^2+bx+12$ ✓subs $S(-2;0)$ and $T(6;0)$ ✓ b-value ✓ $y=-x^2+4x+12$ (4)

	OR $y=a(x-2)^2+q$ $0=a(-2-2)^2+q$ or $0=a(6-2)^2+q$ i.e. $0=16a+q$ $12=a(0-2)^2+q$ i.e. $\underline{12=4a+q}$ $12=-12a$ $a=-1$ $q=16$ $y=-(x-2)^2+16$ $=-(x^2-4x+4)+16$ $=-x^2+4x+12$ **OR** $y=a(x-6)(x+2)$ $=a(x^2-4x-12)$ $=-(x^2-4x-12)$ $=-x^2+4x+12$	✓ $y=a(x-2)^2+q$ ✓ subs R(0 ; 12) and S(– 2 ; 0) (or T(6 ; 0)) ✓ a-value ✓ $y=-x^2+4x+12$ (4) ✓ $y=a(x-6)(x+2)$ ✓ expand ✓ a-value ✓ $y=-x^2+4x+12$ (4)
4.2.3	$\frac{dy}{dx}=0$ $-2x+4=0$ $x=2$ $y=-(2)^2+4(2)+12$ $=16$ TP of f is (2 ; 16) **OR** $x=-\frac{b}{2a}$ $=-\frac{4}{2(-1)}$ $=2$ $y=-(2)^2+4(2)+12$ $=16$ TP of f is (2 ; 16) **OR** $f(x)=-(x-2)^2+16$ TP of f is (2 ; 16)	✓x-value ✓ y-value (2) ✓ x-value ✓ y-value (2) ✓ x-value ✓ y-value (2)

	OR $x = \frac{-2+6}{2}$ $= 2$ $y = -(2)^2 + 4(2) + 12$ $= 16$ TP of f is $(2\ ;\ 16)$	✓ x-value ✓ y-value (2)
4.2.4	$k < 16$ **OR** $(-\infty;16)$	✓✓ answer (2)
4.2.5	Maximum value of $h(x) = 3^{f(x)-12}$ occurs at max value of $f(x)$ Maximum value = 3^{16-12} $= 81$ **OR** Maximum value of $h(x) = 3^{f(x)-12}$ occurs at max value of $f(x)$ $h(2) = 3^{f(2)-12}$ $= 3^{16-12}$ $= 3^4$ or 81 **OR** $f(x) - 12 = -x^2 + 4x$ $= x(4-x)$ which has a maximum value of $f(2) = 4$ $\therefore$ Maximum value of $h(x)$ is 3^4 or 81	✓✓ subs 16 for $f(x)$ ✓ 3^4 or 81 (3) ✓✓ subs 16 for $f(x)$ ✓ 3^4 or 81 (3) ✓✓ subs $f(2) = 4$ ✓ 3^4 or 81 (3) **[20]**

QUESTION 5

5.1	$0 \le x \le 3$ **OR** $[0;3]$ **Note:** if the candidate gives $0 < x < 3$, award 1/2 marks	✓ $0 \le x$ ✓ $x \le 3$ (2)
5.2	f^{-1} : $x = -\sqrt{27y}$ $x^2 = 27y$ $y = \frac{x^2}{27}$ $x \le 0$ OR $(-\infty;0]$	✓ interchange x- and y- values ✓ $y = \frac{x^2}{27}$ ✓ $x \le 0$ or $(-\infty;0)$ (3)
5.3		✓ shape ✓ end at origin ✓ any other point on the graph (3)
5.4	Reflection about the x-axis **OR** $(x\,;y) \rightarrow (x\,;-y)$; $x \ge 0$	✓ answer (1) ✓ answer (1) **[9]**

QUESTION 6

	$f(x)=\frac{a}{x-5}+1$ $0=\frac{a}{(2)-5}+1$ $-1=\frac{a}{-3}$ $a=3$ $f(x)=\frac{3}{x-5}+1$ **OR** $\left(x-5\right)\left(y-1\right)=k$ $\left(2-5\right)\left(0-1\right)=k$ $k=3$ $\left(x-5\right)\left(y-1\right)=3$ $y=\frac{3}{x-5}+1$ **NOTE:** $f\left(x\right)=\frac{x-2}{x-5}$ as an alternative simplified form.	✓ $x-5$ ✓ + 1 ✓ substitution of (2 ; 0) ✓ $a = 3$ (4) ✓ $\left(x-5\right)$ ✓ $\left(y-1\right)$ ✓ substitution of (2 ; 0) ✓ $k=3$ (4) **[4]**

QUESTION 7

7.1.1	$\text{A}=\text{P}(1-\text{i})^{\text{n}}$ $=120\ 000(1-0{,}09)^{5}$ $=\text{R}74\ 883{,}86$ **NOTE:** Incorrect formula (in 7.1.1 or 7.1.2) award max 1/3 marks	✓ *i*, *n* and P identified ✓subs into correct formula ✓ answer (3)
7.1.2	$\text{A}=\text{P}(1+\text{i})^{\text{n}}$ $=120\ 000(1+0{,}07)^{5}$ $=\text{R}168\ 306{,}21$	✓ *i*, *n* and P identified ✓subs into correct formula ✓ answer (3)
7.1.3	Sinking fund needed: $F_v =$ R 90 000 $F_v=\frac{x[(1+i)^n-1]}{i}$ $90\ 000=\frac{x\left[\left(1+\frac{0{,}085}{12}\right)^{61}-1\right]}{\frac{0{,}085}{12}}$ $x=\text{R}\,1\ \ 184{,}68$ **NOTE:** Incorrect formula award max 2/5 marks	✓ $F_v=\text{R 90 000}$ ✓ $i=\frac{0{,}085}{12}=\frac{17}{2400}$ in annuity formula ✓ $n = 61$ ✓ subs into correct formula ✓ answer (5)

	OR Consider the scenario as money deposited at the beginning of every month, but in the last month an additional payment was made at the end of the month: $F_v = \frac{x(1+i)\left[(1+i)^n - 1\right]}{i} + x$ $= x\left(\frac{(1+i)\left[(1+i)^n - 1\right]}{i} + 1\right)$ $90\ 000 = x\left(\frac{\left(1+\frac{0,085}{12}\right)\left[\left(1+\frac{0,085}{12}\right)^{60} - 1\right]}{\frac{0,085}{12}} + 1\right)$ $x = \frac{90\ 000\left(\frac{0,085}{12}\right)}{\left(1+\frac{0,085}{12}\right)\left[\left(1+\frac{0,085}{12}\right)^{60} - 1\right]+\left(\frac{0,085}{12}\right)}$ $= \text{R}1184,68$	✓ $i = \frac{0,085}{12} = \frac{17}{2400}$ in annuity formula ✓ $n = 60$ in annuity formula ✓ $F_v = \text{R } 90\ 000$ ✓ subs into correct formula ✓ answer (5)
	OR Present value of sinking fund needed: $90000 = P_v\left(1+\frac{0,085}{12}\right)^{61}$ $P_v = \text{R}58513,03$ Using the present value formula: $P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $58513,03 = \frac{x\left[1-\left(1+\frac{0,085}{12}\right)^{-61}\right]}{\frac{0,085}{12}}$ $x = \text{R } 1\ 184,68$	✓ $i = \frac{0,085}{12} = \frac{17}{2400}$ in annuity formula ✓ $n = 61$ ✓ $P_v = \text{R}58513,03$ ✓ subs into correct formula ✓ answer (5)

7.2	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $900\,000 = \frac{18\,000\left[1-\left(1+\frac{0{,}105}{12}\right)^{-n}\right]}{\frac{0{,}105}{12}}$ $1-\frac{900\,000\left(\frac{0{,}105}{12}\right)}{18\,000} = \left(1+\frac{0{,}105}{12}\right)^{-n}$ $-n = \log_{\left(1+\frac{0{,}105}{12}\right)}\frac{9}{16}$ $n = 66{,}04$ months She will be able to maintain her current lifestyle for a little more than 66 months using her pension money. **Note:** If F_v formula used, possibly award one each for *x*, *i*, use of logs: max 3/6 marks If any other incorrect formula is used, award 0/6 marks **Note:** If candidate rounds off early in Question 7.2 (and obtain 58 months), penalise 1 mark	✓ x = 18 000 ✓ $i = \frac{0{,}105}{12}$ in annuity formula ✓ subs into correct formula ✓ simplification ✓ use of logs ✓ answer in months (6)
	OR $P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $900\,000 = \frac{18\,000\left[1-\left(1+\frac{0{,}105}{12}\right)^{-n}\right]}{\frac{0{,}105}{12}}$ $1-\frac{900\,000\left(\frac{0{,}105}{12}\right)}{18\,000} = \left(1+\frac{0{,}105}{12}\right)^{-n}$ $-n\log\left(1+\frac{0{,}105}{12}\right) = \log\frac{9}{16}$ $n = 66{,}04$ months She will be able to maintain her current lifestyle for a little more than 66 months using her pension money.	✓ x = 18 000 ✓ $i = \frac{0{,}105}{12}$ in annuity formula ✓ subs into correct formula ✓ simplification ✓ use of logs ✓ answer in months (6)

	OR $A = F_v$ $P(1+i)^n = \frac{x\left[(1+i)^n - 1\right]}{i}$ $900000\left(1+\frac{0,105}{12}\right)^n = \frac{18000\left[\left(1+\frac{0,105}{12}\right)^n - 1\right]}{\frac{0,105}{12}}$ $\frac{0,105}{12}\times 900\,000\left(1+\frac{0,105}{12}\right)^n = 18000\left(1+\frac{0,105}{12}\right)^n - 18000$ $18000 = 18000\left(1+\frac{0,105}{12}\right)^n - \frac{0,105}{12}\times 900\,000\left(1+\frac{0,105}{12}\right)^n$ $18000 = \left(1+\frac{0,105}{12}\right)^n\left(18000 - \frac{0,105}{12}\times 900\,000\right)$ $18000 \div \left(18000 - \frac{0,105}{12}\times 900\,000\right) = \left(1+\frac{0,105}{12}\right)^n$ $\frac{16}{9} = \left(1+\frac{0,105}{12}\right)^n$ $-n = \log_{\left(1+\frac{0,105}{12}\right)}\frac{9}{16}$ $n = 66,04$ months She will be able to maintain her current lifestyle for a little more than 66 months using her pension money.	✓ x = 18 000 ✓ $i = \frac{0,105}{12}$ in annuity formula ✓ subs into correct formula ✓ simplification ✓ use of logs ✓ answer in months (6) **[17]**

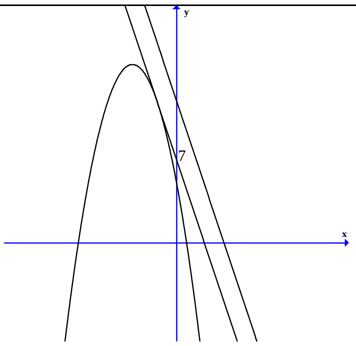
QUESTION 8

8.1	$f(x) = 2x^2 - 5$ $f(x+h) = 2(x+h)^2 - 5$ $= 2x^2 + 4xh + 2h^2 - 5$ $f(x+h) - f(x) = 4xh + 2h^2$ $f'(x) = \lim_{h\to 0}\frac{4xh + 2h^2}{h}$ $= \lim_{h\to 0}\frac{h(4x+2h)}{h}$ $= \lim_{h\to 0}(4x+2h)$ $= 4x$ **OR** **Note:** If candidate makes a notation error Penalise 1 mark **Note:** If candidate uses differentiation rules Award 0/5 marks	✓ substitution of of $x + h$ ✓ simplification to $4xh + 2h^2$ ✓ formula ✓ $\lim_{h\to 0}(4x+2h)$ ✓ answer (5)

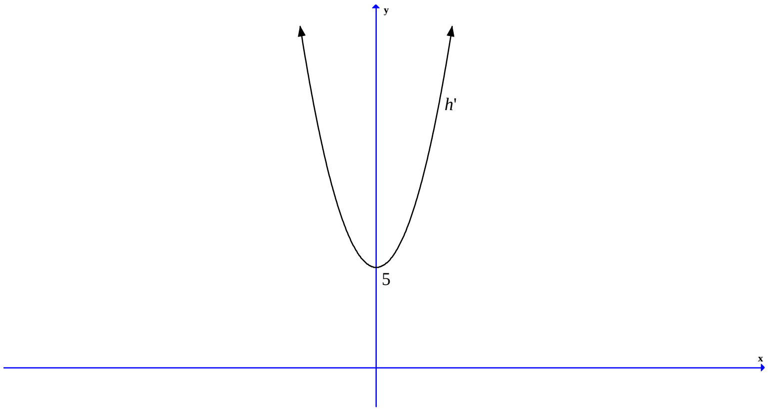
	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{\left[2(x+h)^2-5\right]-\left(2x^2-5\right)}{h}$ $=\lim\limits_{h\to 0}\dfrac{\left[2\left(x^2+2xh+h^2\right)-5\right]-2x^2+5}{h}$ $=\lim\limits_{h\to 0}\dfrac{[2x^2+4xh+2h^2-5]-2x^2+5}{h}$ $=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(4x+2h)}{h}$ $=\lim\limits_{h\to 0}(4x+2h)$ $=4x$	✓ formula ✓ substitution of $x+h$ ✓ simplification to $\dfrac{4xh+2h^2}{h}$ ✓ $\lim\limits_{h\to 0}(4x+2h)$ ✓ answer (5)
8.2	$\dfrac{dy}{dx}=-4x^{-5}+6x^2-\dfrac{1}{5}$ $=\dfrac{-4}{x^5}+6x^2-\dfrac{1}{5}$ **Note:** notation error penalise 1 mark **Note:** candidates do NOT need to give their answer with positive exponents	✓ $-4x^{-5}$ ✓ $6x^2$ ✓ $-\dfrac{1}{5}$ (3)
8.3.1	$g(x)=\dfrac{x^2+x-2}{x-1}$ $=\dfrac{(x+2)(x-1)}{x-1}$ $=x+2 \quad (x\neq 1)$ $g'(x)=1 \quad (x\neq 1)$	✓ simplification ✓ answer (2)
8.3.2	The function is undefined at $x = 1$. **OR** Division by zero is undefined. **OR** The denominator cannot be zero. **OR** In the definition of the derivative , $g'(1)=\lim\limits_{h\to 0}\dfrac{g(1+h)-g(1)}{h}$, but $g(1)$ does not exist.	✓ answer (1) **[11]**

QUESTION 9

9.1.1	$f(x)=-x^3-x^2+16x+16$ $f'(x)=-3x^2-2x+16$ $0=-3x^2-2x+16$ $3x^2+2x-16=0$ $(3x+8)(x-2)=0$ $x=-\frac{8}{3}$ or $x=2$ **Note:** if neither $f'(x)=0$ nor $0=-3x^2-2x+16$ explicitly stated, award maximum 3/4 marks **OR** $f(x)=-x^3-x^2+16x+16$ $f'(x)=-3x^2-2x+16$ $0=-3x^2-2x+16$ $0=3x^2+2x-16$ $x=\frac{-2\pm\sqrt{2^2-4(3)(-16)}}{2(3)}$ $x=-\frac{8}{3}$ or $x=2$	✓ $f'(x)=-3x^2-2x+16$ ✓ $f'(x)=0$ or $0=-3x^2-2x+16$ ✓ factors ✓ x values (4) ✓ $f'(x)=-3x^2-2x+16$ ✓ $f'(x)=0$ or $0=-3x^2-2x+16$ ✓ subs into formula ✓ x values (4)
9.1.2	$f''(x)=0$ $-6x-2=0$ $x=-\frac{1}{3}$ **OR** $x=\frac{-\frac{8}{3}+2}{2}$ $x=-\frac{1}{3}$ **OR** $f'(x)=-3x^2-2x+16$ $x=\frac{-(-2)}{2(-3)}$ $=-\frac{1}{3}$ **OR**	✓ $f''(x)=-6x-2$ ✓ $-6x-2=0$ ✓ answer (3) ✓ $x=\frac{-\frac{8}{3}+2}{2}$ ✓✓ answer (3) ✓✓ $x=\frac{-(-2)}{2(-3)}$ ✓ answer (3)

	$f(x) = -x^3 - x^2 + 16x + 16$ $x = \frac{-(-1)}{3(-1)}$ $= -\frac{1}{3}$	✓✓ $x = \frac{-(-1)}{3(-1)}$ ✓ answer (3)
9.2.1	$g(x) = -2x^2 - 9x + 5$ $g(-1) = -2(-1)^2 - 9(-1) + 5$ $= 12$ $g'(x) = -4x - 9$ $m_{\tan} = -4(-1) - 9$ $= -5$ $y = -5x + c$ $12 = -5(-1) + c$ $c = 7$ $y = -5x + 7$ **OR** $g(x) = -2x^2 - 9x + 5$ $g(-1) = -2(-1)^2 - 9(-1) + 5$ $= 12$ $g'(x) = -4x - 9$ $m_{\tan} = -4(-1) - 9$ $= -5$ $y - 12 = -5(x + 1)$ $y = -5x + 7$	✓ $g(-1) = 12$ ✓ $g'(x) = -4x - 9$ ✓ $m_{\tan} = -5$ ✓ answer (4) ✓ $g(-1) = 12$ ✓ $g'(x) = -4x - 9$ ✓ $m_{\tan} = -5$ ✓ answer (4)
9.2.2	 $q > 7$ **OR** $y = -5x + q$ and $y = -2x^2 - 9x + 5$ $-5x + q = -2x^2 - 9x + 5$ $q = -2(x + 1)^2 + 7$ $\therefore q > 7$	✓ sketch ✓ 7 ✓correct inequality (3) ✓ method ✓ 7 ✓correct inequality (3)

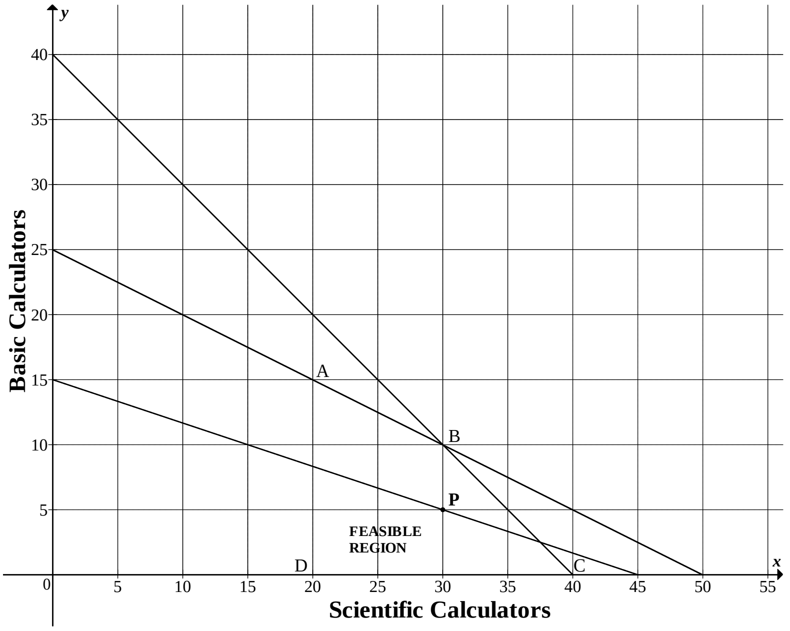
	OR $y=-5x+q$ and $y=-2x^2-9x+5$ $-5x+q=-2x^2-9x+5$ $2x^2+4x+q-5=0$ $x=\dfrac{-4\pm\sqrt{16-4(2)(q-5)}}{2(2)}$ $x=\dfrac{-4\pm\sqrt{56-8q}}{4}$ $56-8q<0$ $q>7$	✓ method ✓ 7 ✓correct inequality (3)
	OR Since $g(-1)=12$ and at $x=-1$, tangent equation is $y=-5x+7$, $y=-5x+q$ not intersecting g $\Rightarrow$ $12<-5(-1)+q$ $12-5<q$ $7<q$	✓ method ✓ 7 ✓correct inequality (3)
9.3	$h'(x)=12x^2+5$ For all values of x: $x^2\geq 0$ $12x^2\geq 0$ $12x^2+5\geq 5$ $12x^2+5>0$ For all values of x: $h'(x)>0$ All tangents drawn to h will have a positive gradient. It will never be possible to draw a tangent with a negative gradient to the graph of *h*.	✓ $h'(x)=12x^2+5$ ✓ clearly argues that $h'(x)>0$ ✓ conclusion (3)
	OR $h'(x)=12x^2+5$ Suppose $h'(x)<0$ and try to solve for x : $12x^2+5<0$ $x^2<-\dfrac{5}{12}$ but x^2 is always positive $\therefore$ no solution for x $\therefore h'(x)\geq 0$ for all $x\in R$ i.e. there are no tangents with negative slopes	✓ $h'(x)=12x^2+5$ ✓ clearly argues that $h'(x)<0$ is impossible ✓ conclusion (3)

	OR	
	$h'(x) = 12x^2 + 5$	✓ $h'(x) = 12x^2 + 5$
	 Since clearly $h'(x) > 0$ for all $x \in R$, it will never be possible to draw a tangent with a negative gradient to the graph of *h*.	✓ argues $h'(x) > 0$ by drawing a sketch ✓ conclusion (3) **[17]**

QUESTION 10

10.1	$s(t) = 2t^2 - 18t + 45$ $s'(t) = 4t - 18$ $s'(0) = 4(0) - 18$ $= -18\ m/s$ **Note:** answer only award 0/3 marks	✓ $s'(t)$ ✓ subs $t = 0$ into $s'(t)$ formula ✓ answer (3)
10.2	$s''(t) = 4\,\text{m/s}^2$	✓ answer (1)
10.3	$4t - 18 = 0$ $4t = 18$ $t = \frac{9}{2}\text{seconds or } 4{,}5\text{seconds}$ **OR** $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$ $t = \frac{9}{2}\text{seconds or } 4{,}5\text{seconds}$ **OR** $s(t) = 2t^2 - 18t + 45$ $t = -\frac{-18}{2(2)}$ $t = \frac{9}{2}\text{seconds or } 4{,}5\text{seconds}$	✓ $s'(t) = 0$ ✓ answer (2) ✓ $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$ ✓ answer (2) ✓ $t = -\frac{-18}{2(2)}$ ✓ answer (2) **[6]**

QUESTION 11



11.1	No, because (15 ; 5) does not lie within the feasible region. **OR** No, because according to the constraints, the *x*-value (number of scientific calculators) must be at least 20.	✓ answer (<u>with</u> motivation) (1)
11.2	$x \geq 20$ $x + 2y \leq 50$ $x + y \leq 40$ $y \geq 0$ **OR** $x \geq 20$ $y \leq -\frac{1}{2}x + 25$ $y \leq -x + 40$ $y \geq 0$ **OR** $x \geq 20$ $\frac{y}{25} + \frac{x}{50} \leq 1$ $\frac{y}{40} + \frac{x}{40} \leq 1$ $y \geq 0$ **OR** $x \geq 20$ $40x + 40y \leq 1600$ $25x + 50y \leq 1250$ $y \geq 0$	✓ $x \geq 20$ ✓✓ $x + 2y \leq 50$ ✓✓ $x + y \leq 40$ ✓ $y \geq 0$ (6)
11.3.1	A	✓ answer (1)
11.3.2	All points on the search line yield the same profit. Hence no such point exists. **OR** If such an (*x* ; *y*) exists, Q = *x* + 3*y* and $y = -\frac{1}{3}x + 15$ so $45 = 3y + x = Q$ Q = 4 500 Hence, there is no such point.	✓✓ No point exists (2) ✓✓ No point exists (2)

11.3.3	$Q = ax + by$ $y = -\frac{a}{b}x + \frac{Q}{b}$ $-1 \le -\frac{a}{b} \le -\frac{1}{2}$ $\frac{1}{2} \le \frac{a}{b} \le 1$ The maximum value of $\frac{a}{b}$ is 1.	✓ $y = -\frac{a}{b}x + \frac{Q}{b}$ ✓ $-1 \le -\frac{a}{b}$ ✓ $\frac{a}{b} \le 1$ ✓ answer (4) **[14]**

TOTAL: 150